V & S PUBLISHERS

Fully Revised

Prasoon Kumar

MATHS OLYMPIAD

International Mathematics Olympiad

6

Strictly according to the latest syllabus of Maths Olympiads

Number

Algebra

Ratio and Proportion

Integers

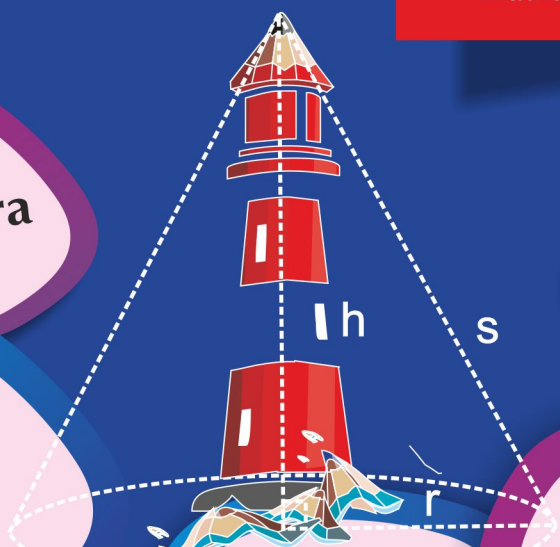
Pattern

Figure Puzzles

Analogy

Geometry

Embedded Figures



The Gen X

A SUCCESS PACKAGE FOR ASPIRANTS OF MATHS OLYMPIAD

IMO

INTERNATIONAL MATHEMATICS OLYMPIAD

6

Prasoon Kumar

V&S PUBLISHERS

Published by:

V&S PUBLISHERS

F-2/16, Ansari road, Daryaganj, New Delhi-110002
☎ 23240026, 23240027 • *Fax:* 011-23240028
Email: info@vspublishers.com • *Website:* www.vspublishers.com

Regional Office : Hyderabad
5-1-707/1, Brij Bhawan (Beside Central Bank of India Lane)
Bank Street, Koti, Hyderabad - 500 095
☎ 040-24737290
E-mail: vspublishershyd@gmail.com

Branch Office : Mumbai
Jaywant Industrial Estate, 1st Floor–108, Tardeo Road
Opposite Sobo Central Mall, Mumbai – 400 034
☎ 022-23510736
E-mail: vspublishersmum@gmail.com

Follow us on:

ISBN 978-93-505795-7-2

DISCLAIMER

PUBLISHER'S NOTE

V&S Publishers, after the grand success of a number of academic and general books, is pleased to bring out a series of *Mathematics Olympiad books* under *The Gen X series – generating Xcellence in generation X* – which has been designed to focus on the problems faced by students. In all books the concepts have been explained clearly through various examples, illustrations and diagrams wherever required. Each book has been developed to meet specific needs of students who aspire to get distinctions in the field of mathematics and want to become Olympiad champs at national and international levels.

To go through Maths Olympiad successfully, students need to do thorough study of topics covered in the *Olympiads syllabus and the topics covered in school syllabus as well*. The Olympiads not only tests the subjective knowledge but Reasoning skills also. So students are required to comprehend the depth of concepts and problems and gain experience through practice. The Olympiads check efficiency of candidates in problem solving. These exams are conducted in different stages at regional, national, and international levels. At each stage of the test, the candidate should be fully prepared to go through the exam. Therefore, this exam requires careful attention towards comprehension of concepts, thorough practice, and application of rules and concepts.

While other books in market focus selectively on questions or theory; V&S Maths Olympiad books are rather comprehensive. Each book has been divided into five sections namely *Mathematics, Logical Reasoning, Achiever's section, Subjective section, and Model Papers*. The theory has been explained through solved examples. To enhance problem solving skills of candidates, *Multiple Choice Questions (MCQs)* with detailed solutions are given at the end of each chapter. Two *Mock Test Papers* have been included to understand the pattern of exam. A CD containing Study Chart for systematic preparation, Tips & Tricks to crack Maths Olympiad, Pattern of exam, and links of Previous Years Papers is accompanied with this book. The books are also useful for various competitive exams such as NTSE, NSTSE, and SLSTSE as well.

We wish you all success in the examination and a very bright future in the field of mathematics.

All the best

CONTENTS

SECTION 1 : MATHEMATICAL REASONING

SECTION 2 : LOGICAL REASONING

Part A : Verbal Reasoning

Part B : Non-Verbal Reasoning

Section 1
Mathematical Reasoning

1 Playing with Numbers

Numbers

We usually write numbers using the Hindu–Arabic system which uses the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

Types of Numbers

1. **Even numbers:** All multiples of 2 are called even numbers, e.g., 2, 4, 6, 8, 10, ….. are all even numbers.
2. **Odd numbers:** Numbers which are not multiples of 2 are called odd numbers, e.g., 1, 3, 5, 7, 9 ….. are all odd numbers.
3. **Perfect numbers:** If the sum of all the factors of a number is two times the number then the number is called a perfect number, e.g. The factors of 6 are 1, 2, 3, and 6.
4. **Co-prime numbers:** Two numbers having only '1' as a common factor are called co-prime numbers, e.g., 5 and 8 are co-prime numbers.

Divisibility Tests of Numbers

1. **Divisibility by 2:** A number is divisible by 2 if its ones digit is 0, 2, 4, 6, or 8, e.g., 62, 60, 198, 294 …. etc.
2. **Divisibility by 3:** A number is divisible by 3 if the sum of its digit is divisible by 3.
3. **Divisibility by 4:** A number is divisible by 4 if the number formed by the last two digits (i.e. ones and tens) of the number is divisible by 4.
4. **Divisibility by 5:** A number is divisible by 5 if the digits in ones places is a 0 or 5.
5. **Divisibility by 7:** A number is divisible by 7 if we follow certain steps to determine this:

   $623 \rightarrow 3 \times 2 = 6$ (Double the digit in one's place).

   $62 - 6 = 56$ (Subtract the number formed by rest of the digits).

   $\because$ 56 is divisible by 7.

   $\therefore$ 623 is divisible by 7.
6. **Divisibility by 9:** A number is divisible by 9 if the sum of its digits is divisible by 9.

Example 1: 1123056 is divisible by

(i) 3 (ii) 5 (iii) 7 (iv) 11

Solution:

(i) Sum of digits = 1 + 1 + 2 + 3 + 0 + 5 + 6 = 18.

$\because$ 18 is divisible by 3.

$\therefore$ 1123056 is divisible by 3.

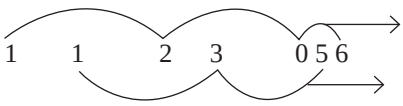
(ii) $\because$ The ones place digit is 6

$\therefore$ The given number is not divisible by 5.

(iii) $112305 - 6 \times 2 = 112305 - 12 = 112293$

$\because$ 112293 is not divisible by 7.

$\therefore$ 1123056 is not divisible by 7.

(iv) 

Sum of digits at odd places = 1 + 2 + 0 + 6 = 9

Sum of digits at even places = 1 + 3 + 5 = 9.

∵ Difference = (9 – 9) = 0

∴ This number is divisible by 11.

Example 2: Check the divisibility of 2345 by 7.

Solution: 234 – 5 × 2 = 224.

∵ 224 is divisible by 7.

∴ 2345 is divisible by 7.

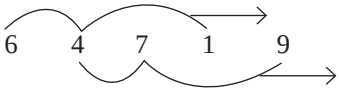
Example 3: Check the divisibility of 9198 by 3 and 9.

Solution: 9 + 1 + 9 + 8 = 27.

∴ the number is divisible by 3 as well as 9.

Example 4: Replace * by the smallest digit, so that it is divisible by 11.

64*719.

Solution: 

Sum of digits on odd places = 6 + * + 1 = 7 + *

Sum of digits on even places = 4 + 7 + 9 = 20

20 – 7 – * = 11λ, where, λ is an integer.

For, λ = 1,

∗ = 2.

Multiples and Factors

Multiple

A number which can be expressed as a product of two or more numbers is called the multiple of those numbers. For example 42 is a multiple of 6 as well as 7.

Factor

A factor is the number which divides the given number completely by leaving remainder '0'. For example 28 is divisible by 1, 2, 4, 7 and 28. So these are factors of 28.

Highest Common Factor

The largest common factor of two or more numbers is their Highest Common Factor (HCF). It is also known as Greatest Common Divisor (GCD).

The possible factors of 42 are 1, 2, 3, 6, 7, 14, 42.

The possible factors of 56 are 1, 2, 4, 7, 8, 14, 28, 56.

∴ The highest common factor of 42 and 56 = 14.

There are two methods to find H.C.F. of given numbers:

1. Prime Factorisation Method,
2. Division Method.

Example 5: Find the H.C.F. of 270 and 729 by prime factorization method.

Solution:

2	270
3	135
3	45
3	15
	5

3	729
3	243
3	81
3	27
3	9
	3

$270 = 2 \times 3 \times 3 \times 3 \times 5$ and

$729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

∴ H.C.F. of 270 and 729 = $3 \times 3 \times 3 = 27$.

Example 6: Find the HCF of 120 and 96 by division method.

Solution:

$$\begin{array}{r} 96\overline{)120}(1 \\ \underline{96} \\ 24\overline{)96}(4 \\ \underline{96} \\ \times\times \end{array}$$

∴ HCF of 120 and 96 = 24.

HCF of Three or More Numbers

Example 7: Find the greatest number which divides 13, 133, 37 leaving 1 as a remainder.

Solution:

13	133	37
−1	−1	−1
12	132	36

Hence, the required number is H.C.F. of 12, 132 and 36.

∴ HCF of 12 and 132

$$\begin{array}{r} 12\overline{)132}(11 \\ \underline{12} \\ 12 \\ \underline{12} \\ \times\times \end{array}$$

HCF of 12 and 132 = 12.

Similarly, HCF of 12 and 36 = 12.

∴ HCF of 12, 132, and 36 = 12.

∴ Required number = 12.

Lowest Common Multiple (LCM)

The lowest common multiple of two or more numbers is the smallest number which is multiple of each of the given numbers.

There are two methods to find LCM of given numbers.

(i) Prime factorisation method

(ii) Division method

Example 8: Find the LCM of 4, 16, 20, 24 and 36.

Solution:

2	4, 16, 20, 24, 36
2	2, 8, 10, 12, 18
2	1, 4, 5, 6, 9
3	1, 2, 5, 3, 9
	1, 2, 5, 1, 3

∴ LCM of 4, 16, 20, 24 and 36 = 2 × 2 × 2 × 3 × 2 × 5 × 3 = 1440.

Example 9: Four bells toll at intervals 5, 10, 15 and 25 seconds. The bells toll together of 6 O'clock. When will they toll together again?

Solution: Time when the bells will toll together = LCM of 5, 10, 15 and 25.

5	5, 10, 15, 25
	1, 2, 3, 5

LCM of 5, 10, 15 and 25 = 5 × 2 × 3 × 5

= 150 seconds.

∴ 150 seconds = 2.5 min.

∴ Bells will again toll 2.5 minutes past 6 o'clock.

Relation between HCF and LCM of Two Numbers

For any two numbers '*x*' and '*y*',

Product of *x* and *y* = HCF of *x* and *y* × LCM of *x* and y.

Example 10: The HCF of two numbers is 4 and their product is 288. Find the LCM of two numbers.

Solution: LCM × HCF = Product of numbers.

$$\therefore \text{LCM} = \frac{\text{product of numbers}}{\text{HCF}}$$

$$= \frac{288}{4}$$

$$= 72$$

Multiple Choice Questions

1. The HCF of two numbers is 16 and their product is 3072. What is the LCM?
   (a) 182 (b) 162
   (c) 192 (d) 196
2. Which of the following numbers is divisible by 3?
   (a) 24357806 (b) 33336433
   (c) 35769812 (d) 83479560
3. Which of the following is a prime number?
   (a) 117 (b) 171
   (c) 179 (d) 169
4. Which of the following is not a twin prime?
   (a) (11, 13) (b) (17, 19)
   (c) (23, 29) (d) (41, 43)
5. Find the greatest number which divides 126, 150 and 210 leaving remainder 6 in each case.
   (a) 12 (b) 14
   (c) 16 (d) 22
6. Find the largest number that will divide 76, 113 and 186 leaving remainder 4, 5, 6 respectively.
   (a) 24 (b) 12
   (c) 36 (d) 54
7. Find the smallest number which when divided by 16, 36 & 40 leaves a remainder 7 in each case.
   (a) 627 (b) 727
   (c) 827 (d) 927
8. Which greatest number of 4 digits is exactly divisible by 12, 16, 28 & 36?
   (a) 6072 (b) 8072
   (c) 8972 (d) 9072
9. The HCF of two numbers is 23 and their LCM is 1449. If one of the numbers is 161 what is the other?
   (a) 107 (b) 117
   (c) 167 (d) 207
10. Find the smallest number which when diming by 3 is divisible by 21, 28, 36 and 45?
    (a) 1163 (b) 1263
    (c) 1283 (d) 1293
11. The HCF of two numbers is 145 and their LCM is 2175. If one of the numbers is 725. What is the other number?
    (a) 5 (b) 290
    (c) 115 (d) 435
12. Which of the following is a composite number?
    (a) 23 (b) 29
    (c) 32 (d) 41
13. Which longest tape can be used to measure exactly the length 7m, 3m 85cm and 12m 95 cm?
    (a) 45 cm (b) 35 cm
    (c) 105 cm (d) 70 cm
14. The greatest number that will divide 445, 572 & 699 leaving remainder 4, 5, 6 respectively.
    (a) 84 (b) 42
    (c) 49 (d) 63
15. What is the sum of LCM & HCF of 1152 & 1664?
    (a) 14976 (b) 15104
    (c) 15114 (d) 15204
16. The HCF of two numbers is 21 and their LCM is 3003. If one of the numbers is 231. Then what is the other number?
    (a) 273 (b) 263
    (c) 283 (d) 293
17. Find the greatest 3-digit number which is divisible by 8, 10 and 12.
    (a) 840 (b) 480
    (c) 960 (d) 980
18. Which of the following number is not divisible by 9?
    (a) 387459 (b) 904806
    (c) 758934 (d) 879134

19. Find the smallest possible number which on adding 19 becomes exactly divisible by 28, 36 and 45.
(a) 1239 (b) 1241
(c) 1243 (d) 1245

20. Four bells toll at intervals 4, 7, 12 & 84 seconds. The bells toll together at 7 o'clock. How many times will they again toll together in 28 minutes?
(a) 15 (b) 20
(c) 25 (d) 30

21. What is the least 5-digit number which is exactly divisible by 20, 25, 30?
(a) 10100 (b) 10200
(c) 10300 (d) 10400

22. What is the maximum even multiple of 25 between 500 & 700?
(a) 660 (b) 600
(c) 675 (d) 650

23. Which of the following number is divisible by 8?
(a) 162537 (b) 764918
(c) 825908 (d) 694728

24. Which of the following is divisible by 11?
(a) 65483 (b) 72493
(c) 84527 (d) 92056

25. What is sum of first five multiples of 23?
(a) 341 (b) 342
(c) 343 (d) 345

26. Which of the following statement is true?
(a) 1509344 is divisible by 8.
(b) 72493 is divisible by 11.
(c) 8569 is not divisible by 11.
(d) 115 is a multiple of 19.

27. In 467 * 381 replace * by which smallest digit to make it divisible by 3?
(a) 1 (b) 2
(c) 3 (d) 4

28. 1870 is divisible by 22. Which two numbers nearest to 1870 are each divisible by 22?
(a) 1848, 1892 (b) 1893, 1914
(c) 1826, 1914 (d) None of these

29. There are three heaps of rice weighing 120 kg, 144 kg and 204 kg. What is the maximum capacity of a bag so that the rice of each can be packed in exactly number of bags?
(a) 24 kg (b) 18 kg
(c) 12 kg (d) 6 kg

30. Four bells ring at intervals of 6, 8, 12 & 20 minutes. They ring simultaneously at 7 a.m. At what time will they ring together?
(a) 8 a.m. (b) 9 a.m.
(c) 10 a.m. (d) 9:30 a.m.

Answer Key

1. (c)	2. (d)	3. (c)	4. (c)	5. (a)	6. (c)	7. (b)	8. (d)	9. (d)	10 (b)
11. (d)	12. (c)	13. (b)	14. (d)	15. (b)	16. (a)	17. (c)	18. (d)	19. (b)	20. (b)
21. (a)	22.(d)	23. (d)	24. (a)	25. (d)	26. (a)	27. (a)	28. (a)	29. (c)	30. (b)

1. **(c)** HCF × LCM = product of numbers.

$\Rightarrow$ LCM = $\frac{\text{Product of numbers}}{\text{HCF}}$

$= \frac{3072}{16} = 192$

2. **(d)**

2 + 4 + 3 + 5 + 7 + 8 + 0 + 6 = 35

3 + 3 + 3 + 3 + 6 + 4 + 3 + 3 = 28

3 + 5 + 7 + 6 + 9 + 8 + 1 + 2 = 41

8 + 3 + 4 + 7 + 9 + 5 + 6 + 0 = 42

Since 42 is divisible by 3 then (d) is divisible by 3.

3. **(c)**

117, 171 are divisible by 3.

169 is divisible by 13.

179 is a prime number.

4. **(c)**

(23, 29) is not a twin prime.

5. **(a)**

126	150	210
− 6	− 6	− 6
120	144	204

HCF of 120 and 144 = 12

HCF of 120 and 204 = 12

Required number = HCF of 120, 144, 204

= 12

6. **(c)**

76	113	186
−4	−5	− 6
72	108	180

HCF of 72 and 108 = 36

HCF of 108 and 180 = 36

$\therefore$ required number = 36

7. **(b)**

LCM of 16, 36 and 40 is

2	16, 36, 40
2	8, 18, 20
2	4, 9, 10
	2, 9, 5

$\therefore$ LCM = 2 × 2 × 2 × 2 × 9 × 5

= 16 × 45 = 720

$\therefore$ required number = (720 + 7) = 727

8. **(d)**

The greatest 4 digit number = 9999.

Here,

2	12, 16, 28, 36
2	6, 8, 14, 18
2	3, 4, 7, 9
	3, 2, 7, 3

LCM of 12, 16, 28 and 36

= 2 × 2 × 2 × 3 × 2 × 7 × 3

= 4 × 9 × 4 × 7

= 16 × 9 × 7

= 1008

$\because$ 9072 is a multiple of 1008.

$\therefore$ it is the greatest number of 4 digits which is divisible by 12, 16, 28 & 36.

9. **(d)**

LCM × HCF = one number × second number

$\Rightarrow$ Second number $= \frac{23 \times 1449}{161}$

$= 23 \times 9$

$= 207$

10. **(b)**

LCM of 21, 28, 36 and 45 is

3	21, 28, 36, 45
2	7, 28, 12, 15
3	7, 14, 6, 15
2	7, 14, 2, 5
7	7, 7, 1, 5
	1, 1, 1, 5

∴ LCM of 21, 28, 36 and 45

$= 3 \times 2 \times 3 \times 2 \times 7 \times 5$

$= 36 \times 35$

$= 1260$

∴ Required number = 1260 + 3 = 1263

11. **(d)**

$$\text{Required number} = \frac{\text{HCF} \times \text{LCM}}{\text{one of the numbers}}$$

$$= \frac{145 \times 2175}{725}$$

$= 435$

12. **(c)**

∵ 32 has 1, 2, 4, 8, 16, 32, as its factors.

∴ 32 is a composite number.

13. **(b)**

Required length = HCF of 700, 385 and 1295

= 35cm

14. **(d)**

445 − 4 = 441 ; 572 − 5 = 567 ; 699 − 6 = 693

3	441
3	147
7	49
	7

3	567
3	189
3	63
3	21
	7

3	693
3	231
7	77
	11

HCF of 441, 567 and 693 = 3 × 3 × 7

= 63

∴ Required number = 63

15. **(b)**

HCF of 1152 and 1664

2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
	3

2	1664
2	832
2	416
2	208
2	104
2	52
2	26
	13

∴ HCF of 1152 and 1664 = 2^7 = 128

and LCM of 1152 and 1664 = 14976

2	1152, 1664
2	526, 832
	263, 416

∴ LCM + HCF = 14976 + 128 = 15104.

16. **(a)**

$$\text{Required number} = \frac{21 \times 3003}{231}$$

$= 273$

17. **(c)**

LCM of 8, 10 and 12

2	8, 10, 12
2	4, 5, 6
	2, 5, 3

∴ LCM of 8, 10, 12 = 2 × 2 × 2 × 5 × 3

= 120

∴ Greatest 3-digit number which is divisible by 8, 10 and 12 = 120 × 8 = 960

18. **(d)**

3 + 8 + 7 + 4 + 5 + 9 = 36

9 + 0 + 4 + 8 + 0 + 6 = 27

7 + 5 + 8 + 9 +3 + 4 = 36

8 + 7 + 9 + 1 + 3 + 4 = 32

∵ 32 is not divisible by 9.

∴ 879134 is not divisible by 9.

19. **(b)**

LCM of 28, 36 and 45

2	28, 36, 45
2	14, 18, 45
3	7, 9, 45
3	7, 3, 15
	7, 1, 5

∴ LCM of 28, 36 and 45

$= 2 \times 2 \times 3 \times 3 \times 7 \times 5$

$= 4 \times 9 \times 35$

$= 140 \times 9$

$= 1260$

∴ Required number = 1260 – 19

= 1241

20. **(b)**

LCM of 4, 7, 12 and 84 =

2	4, 7, 12, 84
2	2, 7, 6, 42
3	1, 7, 3, 21
7	1, 7, 1, 7
	1, 1, 1, 1

$= 2 \times 2 \times 3 \times 7 = 84$ seconds.

∴ The bells will toll together after 84 seconds.

∴ Number of times the bells will toll together in 28 minutes $= \frac{28 \times 60}{84} = 20$

21. **(a)**

LCM of 20, 25, 30 is

2	20, 25, 30
5	10, 25, 15
	2, 5, 3

∴ LCM of 20, 25, 30 $= 2 \times 5 \times 2 \times 5 \times 3$

$= 300$

∴ least 5-digit number which is exactly divisible by 20, 25, 30 = 10100.

22. **(d)**

Multiples of 25 between 500 and 700 = 525, 550, 575, 600, 625, 650, 675.

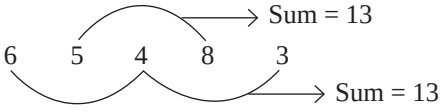
∴ Required number = 650

23. **(d)**

694728 is divisible by 2 as well as 4.

∴ 694728 is divisible by 8.

24. **(a)**



Difference = 13 – 13 = 0

∴ 65483 is divisible by 11.

25. **(d)**

Sum of first five multiples of 23 = 23 + 23 × 2 + 23 × 3 + 23 × 4 + 23 × 5

= 23 (1 + 2 + 3 + 4 + 5)

= 23 × 15

= 345

26. **(a)**

1509344 is divisible by 8.

27. **(a)**

467 * 381

The sum of digits of the above number

= 4 + 6 + 7 + * + 3 + 8 + 1

= 10 + 7 + * + 3 + 8 + 1

= 29 + *

If, * = 1, then, the sum of digits will become 30.

∴ required number = 1

28. **(a)**

1870 + 22 = 1892

1870 – 22 = 1848

29. **(c)**

Required capacity of bag

= HCF of 120, 144 and 204

= 12 kg

30. **(b)**

Required time = LCM of 6, 8, 12 and 20

2	6, 8, 12, 20
2	3, 4, 6, 10
3	3, 2, 3, 5
	1, 2, 1, 5

∴ Required time = $2 \times 2 \times 3 \times 2 \times 5$

= 120 minutes = 2 hr.

∴ bells will again ring together at (7 + 2)

= 9 a.m.