

Jianye Hao · Ho-fung Leung

# Interactions in Multiagent Systems: Fairness, Social Optimality and Individual Rationality



Higher  
Education  
Press



Springer

# Interactions in Multiagent Systems: Fairness, Social Optimality and Individual Rationality



Jianye Hao • Ho-fung Leung

# Interactions in Multiagent Systems: Fairness, Social Optimality and Individual Rationality



Higher  
Education  
Press



Springer

Jianye Hao  
School of Computer Software  
Tianjin University  
Tianjin, China

Ho-fung Leung  
Department of Computer Sci. and Eng.  
The Chinese University of Hong Kong  
Hong Kong, China

ISBN 978-3-662-49468-4  
HEP ISBN 978-7-04-044111-6  
DOI 10.1007/978-3-662-49470-7

ISBN 978-3-662-49470-7 (eBook)

Library of Congress Control Number: 2016933452

© Higher Education Press, Beijing and Springer-Verlag Berlin Heidelberg 2016

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Printed on acid-free paper

This Springer imprint is published by Springer Nature  
The registered company is Springer-Verlag GmbH Berlin Heidelberg

# Preface

It has long been a fundamental assumption in the research of autonomous agents and multiagent systems that intelligent agents should be rational—well, at least as rational as possible even if they are handicapped, like lacking access to all information that is needed for rational decision-making. While this assumption is only natural and right, rationality is actually only a better word to use if we try to avoid the use of the word selfishness to describe the deed of concerning only with one's own interests and being regardless of that of others.

When we first decided to work on the issues of fairness and social optimality in multiagent systems, with due respect to individual rationality of agents, we knew that we were facing a nontrivial challenge. The first difficulty that we encountered was the definition of fairness. We did spend some time to investigate this issue, delving into the definitions given by various authoritative dictionaries, hoping to obtain some mathematical formalism that captured the essence of these definitions. Unfortunately, the definitions in most of these dictionaries were not too helpful to us. While all our efforts seemed to have ended up in vain, we found that we had no choice but to start by taking a simplistic approach to reduce fairness to equality of agents' utility values (in some cases with a sufficiently small difference). Such an approach suffers from several drawbacks, as it is not able to capture many real-life situations. For example, in real life some people might consider it fair for rich people to receive less and poor people to receive more concerning the allocation of public resources. To this end, we looked into fairness models considering other types of fairness such as inequity-aversion and reciprocity. We hope the work presented in this book can shed light on this research area, which could attract more people to continue to work on it.

In this book we aim at tackling the issues of fairness and social optimality in multiagent systems from several aspects. We start with a relatively simple setting, in which agents use adaptive periodical strategy for coordination toward fairness. Agents interact with one another and review and adjust their strategies periodically. We then proceed to investigate how agents can coordinate to social optimality with reinforcement social learning in various types of games. After that, we discuss a related scenario of automated agent negotiation and present a

negotiation strategy that aims at achieving a mutually beneficial agreement. We also considered a competitive negotiation problem and present an adaptive negotiation strategy enabling a rational agent to maximize his benefits from negotiation. Finally, we propose the novel idea of decision entrustment in two-agent repeated games—that is, an agent is allowed the option of entrusting his opponent to make joint decisions in bilateral interaction—and show that social optimality sustained by Nash equilibrium can be achieved.

There are several assumptions that we employ in the work presented in this book. The first one is that agents can always learn from their experiences, and the learning is always rational. That is to say, if an agent, through his experience, finds that an action is most likely to bring its best utility under certain conditions, then the agent should in principle use that action when conditions are met. This is most appropriate when the environment is cooperative. Second, agents should be able to observe the other agents—or some of them—in the system, including the other agents' actions and even the utilities they obtain through interaction. Last but not least, we base our work on some principles proposed by cognitive psychologists. For example, we use a theory by Fehr and Schmidt that people are inequality averse under certain conditions, as well as the research result due to Dawes and Thaleri that people are willing to be kind to those who are being kind and harsh to those who are being unkind. These research works provide a solid cognitive psychological basis for many of the assumptions used throughout this book.

Finally, we would like to express our gratitude for all the help from Higher Education Press and Springer in the publication of this book.

Tianjin, China  
Hong Kong, China  
August 2015

Jianye Hao  
Ho-fung Leung

# Contents

<b>1 Introduction</b> .....	1
1.1 Overview of the Chapters .....	2
1.2 Guide to the Book .....	3
References .....	5
<b>2 Background and Previous Work</b> .....	7
2.1 Background .....	7
2.1.1 Single-Shot Normal-Form Game .....	7
2.1.2 Repeated Games .....	9
2.2 Cooperative Multiagent Systems .....	10
2.2.1 Achieving Nash Equilibrium .....	10
2.2.2 Achieving Fairness .....	13
2.2.3 Achieving Social Optimality .....	16
2.3 Competitive Multiagent Systems .....	19
2.3.1 Achieving Nash Equilibrium .....	19
2.3.2 Maximizing Individual Benefits .....	20
2.3.3 Achieving Pareto-Optimality .....	21
References .....	23
<b>3 Fairness in Cooperative Multiagent Systems</b> .....	27
3.1 An Adaptive Periodic Strategy for Achieving Fairness .....	28
3.1.1 Motivation .....	28
3.1.2 Problem Specification .....	30
3.1.3 An Adaptive Periodic Strategy .....	32
3.1.4 Properties of the Adaptive Strategy .....	36
3.1.5 Experimental Evaluations .....	40
3.2 Game-Theoretic Fairness Models .....	46
3.2.1 Incorporating Fairness into Agent Interactions Modeled as Two-Player Normal-Form Games .....	46
3.2.2 Incorporating Fairness into Infinitely Repeated Games with Conflicting Interests for Conflict Elimination ...	56
References .....	69



<b>4</b>	<b>Social Optimality in Cooperative Multiagent Systems</b> .....	71
4.1	Reinforcement Social Learning of Coordination in Cooperative Games .....	72
4.1.1	Social Learning Framework .....	73
4.1.2	Experimental Evaluations .....	77
4.2	Reinforcement Social Learning of Coordination in General-Sum Games .....	82
4.2.1	Social Learning Framework .....	82
4.2.2	Analysis of the Learning Performance Under the Social Learning Framework .....	88
4.2.3	Experimental Evaluations .....	89
4.3	Achieving Socially Optimal Allocations Through Negotiation .....	100
4.3.1	Multiagent Resource Allocation Problem Through Negotiation .....	101
4.3.2	The APSOPA Protocol to Reach Socially Optimal Allocation .....	102
4.3.3	Convergence of APSOPA to Socially Optimal Allocation .....	108
4.3.4	Experimental Evaluation .....	110
	References .....	112
<b>5</b>	<b>Individual Rationality in Competitive Multiagent Systems</b> .....	115
5.1	Introduction .....	115
5.2	Negotiation Model .....	117
5.3	<i>ABiNeS</i> : An Adaptive Bilateral Negotiating Strategy .....	119
5.3.1	Acceptance-Threshold (AT) Component .....	121
5.3.2	Next-Bid (NB) Component .....	122
5.3.3	Acceptance-Condition (AC) Component .....	124
5.3.4	Termination-Condition (TC) Component .....	125
5.4	Experimental Simulations and Evaluations .....	125
5.4.1	Experimental Settings .....	126
5.4.2	Experimental Results and Analysis: Efficiency .....	128
5.4.3	Detailed Analysis of <i>ABiNeS</i> Strategy .....	130
5.4.4	The Empirical Game-Theoretic Analysis: Robustness .....	133
5.5	Conclusion .....	141
	References .....	141
<b>6</b>	<b>Social Optimality in Competitive Multiagent Systems</b> .....	143
6.1	Achieving Socially Optimal Solutions in the Context of Infinitely Repeated Games .....	143
6.1.1	Learning Environment and Goal .....	144
6.1.2	TaFSO: A Learning Approach Toward SOSNE Outcomes .....	147
6.1.3	Experimental Simulations .....	152
6.2	Achieving Socially Optimal Solutions in the Social Learning Framework .....	158
6.2.1	Social Learning Environment and Goal .....	159
6.2.2	Learning Framework .....	161

6.2.3	Experimental Simulations .....	164
References	.....	169
<b>7</b>	<b>Conclusion</b> .....	171
Reference	.....	173
<b>A</b>	<b>The 57 Structurally Distinct Games</b> .....	175

# Chapter 1

## Introduction

Multiagent systems (MASs) have become a commonly adopted paradigm to model and solve real-world problems. Many competing definitions exist for a MAS. In this book, we consider a typical MAS as a system involving multiple autonomous software agents (or humans) interacting with each other with (possibly) conflicting interests and limited information, and the payoff of each agent is determined by the joint actions of all (or some) agents involved. Therefore, different from single-agent environments, in multiagent interaction environments, each agent needs to take other agents' behaviors into consideration when it makes its own decisions, since others' behaviors can directly influence what it expects from the system. The major question we seek to answer in this book can be summarized as follows: how can a *desirable goal* be achieved in different *multiagent interaction environments* where each agent may have its own limitations and (possibly) conflicting interests?

We divide different multiagent interaction environments into two major categories depending on the underlying intentional stance of the agents: cooperative multiagent environment and competitive multiagent environment. We say a multiagent interaction environment is cooperative if (most of) the agents within the system are willing to behave exactly as they are asked to even if this would cause conflicts with their individual interests. In other words, each agent is cooperative in achieving a commonly shared system-level goal even at the cost of its personal benefits. A multiagent interaction environment is considered to be competitive when each agent is only interested in maximizing its individual benefits. Thus, in competitive environments, agents may not have the incentive to follow what the system designer specifies and usually behave in purely individually rational manners.

We mainly focus on three major goals: fairness, social optimality, and individual rationality. To achieve either of the above goals, the research focus may vary and different approaches may be required depending on the nature of multiagent interaction problems being considered. In cooperative environments, since the agents are assumed to comply with the system designer's instructions, we usually need to consider how we can design an efficient learning strategy through which

the agents can effectively coordinate their behaviors toward the goal we target at (e.g., fairness, social optimality) subject to practical limitations (e.g., limited information available, limited computational, and communication ability). On the contrary, in competitive environments, we mainly focus on addressing the following two questions:

- How can an agent obtain as much individual benefits as possible in the presence of other selfish agents?
- The second one is from the perspective of mechanism design: how can selfish agents be incentivized to autonomously adjust their behaviors toward the goal (e.g., social optimality) we target at?

To model the strategic interactions in MASs, the most powerful and commonly used mathematical tool is game theory. It provides a rich set of powerful and convenient mathematical frameworks to model different forms of strategic interactions among agents. For example, normal-form games can be used to model the single-shot strategic interactions among agents in which each agent makes decision simultaneously; for finitely/infininitely repeated games, which consists of finite/infinite repetition of stage games, they are naturally suitable for modeling the repeated (with unknown rounds) interactions among agents. Thus, in this book, we mainly study the problem of achieving different desirable goals by modeling agents' interactions as different types of games.

## 1.1 Overview of the Chapters

The book is organized in terms of the goals (fairness, social optimality, and individual rationality) we target at and the characteristic of the multiagent interaction environment (cooperative or competitive) we look at. Specifically, we focus on the following four parts, each of which corresponds to one chapter.

**Fairness in cooperative MASs** We pursue the goal of fairness in cooperative MASs from the following two perspectives. First, we describe an adaptive periodical strategy to enable the agents to coordinate toward fair and efficient outcomes in conflicting-interest games [1]. Second, two game-theoretic fairness models are described for two important multiagent interaction frameworks. The first is a game-theoretic fairness model for infinitely repeated game with conflict interest, in which it can be proved that there always exist fairness equilibria under which fair and optimal outcomes can be achieved [2]. The second is a game-theoretic fairness model for single-shot normal-form games, which can better explain actual human behaviors [3].

**Toward social optimality in cooperative MASs** In cooperative MASs, we describe two different types of learners for cooperative games, independent action learners and joint action learners, to coordinate on socially optimal outcomes under the social learning framework [4–6]. For general-sum games,

we introduce a learning framework under which the agents can effectively learn to coordinate on socially optimal outcomes by adaptively choosing between individual learning and social learning [7]. Last, we turn to look at the practical bilateral negotiation problem and introduce an efficient negotiation framework for agents to achieve socially optimal allocations in cooperative bilateral negotiation environments [8].

**Maximizing individual benefits in competitive MASs** In competitive MASs, we focus on the bilateral negotiation setting in which two agents negotiate with each other with the goal of maximizing their individual utilities through negotiation. We describe an adaptive negotiation *ABiNeS* strategy which empirically shows to be very effective in competing with the state-of-the-art negotiation strategies [9, 10]. It is worth mentioning that the implementation of this *ABiNeS* strategy wins the champion of the third international Automated Negotiating Agents Competition [11] in both the qualifying and final rounds.

**Achieving social optimality in competitive MASs** In competitive MASs, we, as the system designer, may be also interested in achieving social optimality. However, in competitive environments, the agents are individually rational and interested in maximizing their own utilities only, which may have conflicts with the goal of social optimality. To resolve this conflict, we introduce an incentive mechanism based on sequential play to enable rational agents to have the motivation to adopt the policy of achieving socially optimal outcomes, which are successfully applied in two different learning frameworks [12–14].

## 1.2 Guide to the Book

The intended reader of this book can be a graduate student or an advanced undergraduate or any researcher in the areas of computer science, game theory, and any other related areas. The material of each chapter is independently organized and the readers can have no difficulty of understanding the materials by jumping to any interested chapter freely. We outline the chapters that follow.

- Chapter 2 first reviews some background concepts in game theory which will be used throughout the book and then gives an overall review of previous work in multiagent learning literature, which are divided into two major categories: cooperative multiagent systems and competitive multiagent systems. The previous works within each category are further divided according to the solution concepts they adopt, and how they link with the works in this book is also mentioned.
- Chapter 3 focuses on investigating the question of how to achieve fairness within cooperative multiagent environments. We interpret fairness from the following two perspectives. First, we focus on the multiagent interaction scenarios with conflict interest and our goal is to achieve fairness among agents in terms of equalizing the payoffs of agents as much as possible. Second, our human beings are not purely selfish but care about fairness, which cannot be explained and

predicted by the classical game theory. Thus, it is worthwhile to develop fairness-based game-theoretic model to better explain and predict human behaviors. Overall, we investigate the goal of fairness in MAS from two perspectives: strategy design and fairness game-theoretic model design, which are introduced in Sects. 3.1 and 3.2, respectively.

- Chapter 4 turns to the investigation of how to achieve social optimality within cooperative multiagent environments. Achieving social optimality means maximizing the sum of all agents' payoff involved in the interaction. In cooperative multiagent environments, the agents are assumed to be willing to cooperate toward social optimality, but the limited information available to each agent might impede effective coordination toward socially optimal outcomes among them. We model the agent interactions using two different types of games, cooperative games and general-sum games, and describe the corresponding state-of-the-art social learning frameworks toward social optimality in Sects. 4.1 and 4.2, respectively. Lastly we consider a particular multiagent negotiation problem and describe an efficient negotiation mechanism for achieving socially optimal allocations in Sect. 4.3.
- Chapter 5 focuses on the problem of maximizing an agent's individual benefits within competitive multiagent systems. In competitive multiagent systems, each agent is usually only interested in maximizing its individual benefits, and a natural question from a single agent's perspective is how to design an efficient strategy to obtain as much payoff as possible by exploiting its partners during interactions. Therefore, in this chapter, we consider the competitive bilateral negotiation environment and introduce a state-of-the-art efficient negotiation strategy to maximize the individual utility of a negotiating agent against other competitive opponents.
- Chapter 6 is devoted to investigating how to achieve social optimality (maximizing system level's performance) within competitive multiagent systems. As we mentioned before, in competitive multiagent systems, the agents may not have the incentive to cooperate toward the goal of social optimality which may conflict with its individual benefits. Therefore, from the system designer's perspective, we need to design efficient mechanism to incentivize selfish agents to cooperate toward socially optimal outcomes. Therefore, in this chapter, we introduce an interesting variation of sequential play and show how this can be applied in different competitive multiagent interaction environments to facilitate agents to achieve socially optimal outcomes without undermining their individual rationality and autonomy, namely, two-agent repeated interaction framework (Sect. 6.1) and social learning framework (Sect. 6.2).
- Chapter 7 concludes the book with a summary of the book and also points out some future research directions for the readers.

## References

1. Hao JY, Leung HF (2010) Strategy and fairness in repeated two-agent interaction. In: Proceedings of ICTAI'10, Arras, pp 3–6
2. Hao JY, Leung HF (2012) Incorporating fairness into infinitely repeated games with conflicting interests for conflicts elimination. In: Proceedings of ICTAI'12, Athens, pp 314–321
3. Hao JY, Leung HF (2012) Incorporating fairness into agent interactions modeled as two-player normal-form games. In: Proceedings of PRICAI'12, Kuching
4. Hao JY, Leung HF (2013) Reinforcement social learning of coordination in cooperative multi-agent systems(extended abstract). In: Proceedings of AAMAS'13, St. Paul, pp 1321–1322
5. Hao JY, Leung HF (2013) The dynamics of reinforcement social learning in cooperative multiagent systems. In: Proceedings of IJCAI'13, Beijing, pp 184–190
6. Hao JY, Leung HF, Ming Z (2014) Multiagent reinforcement social learning toward coordination in cooperative multiagent systems. *ACM Trans Auton Adapt Syst* 9(4):20
7. Hao JY, Leung HF (2011) Learning to achieve social rationality using tag mechanism in repeated interactions. In: Proceedings of ICTAI'11, Boca Raton, pp 148–155
8. Hao JY, Leung HF (2012) An efficient negotiation protocol to achieve socially optimal allocation. In: PRIMA 2012: principles and practice of multi-agent systems, Kuching
9. Hao JY, Leung HF (2012) Abines: an adaptive bilateral negotiating strategy over multiple items. In: Proceedings of IAT'12, Macau, vol 2, pp 95–102
10. Hao JY, Song SZ, Leung HF, Ming Z (2014) An efficient and robust negotiating strategy in bilateral negotiations over multiple items. *Eng Appl Artif Intell* 34:45–57
11. Marsa-Maestre I, Lopez-Carmona MA, Ito T et al (2014) Novel insights in agent-based complex automated negotiation[M]. Springer, Tokyo
12. Hao JY, Leung HF (2012) Learning to achieve socially optimal solutions in general-sum games. In: PRICAI 2012: trends in artificial intelligence, Kuching, pp 88–99
13. Hao JY, Leung HF (2012) Achieving social optimality with influencer agents. In: Proceedings of complex'12, Santa Fe
14. Hao JY, Leung HF (2015) Introducing decision entrustment mechanism into repeated bilateral agent interactions to achieve social optimality. *Auton Agents Multi-Agent Syst* 29(4):658–682

# Chapter 2

## Background and Previous Work

In this chapter, we first review some concepts and terminologies from game theory and multiagent systems areas that will be used throughout the book in Sect. 2.1. After that, we explore previous work in multiagent learning literature by dividing them into two major categories: cooperative multiagent systems and competitive multiagent systems, distinguished by the underlying intentional stance of the agents within the system. Within each category, we review previous work in three different parts distinguished by the different goal that the work targets at. In Sect. 2.2, we focus on reviewing previous work whose goal falls into one of the following major solution concepts: Nash equilibrium, fairness, or social optimality. In Sect. 2.3, we focus on investigating previous work targeting at either of the following major solution concepts: Nash equilibrium, maximizing individual benefits, and Pareto-optimality.

### 2.1 Background

Game theory has been the most commonly adopted mathematical tool for us to model the strategic interactions among different agents in different interaction scenarios. We usually model the one-time interaction among agents as single-shot normal-form game and finite/infinite repeated games to model the repeated interaction among agents.

#### 2.1.1 *Single-Shot Normal-Form Game*

In single-shot games, each agent is allowed to choose one action from its own action set simultaneously, and each agent will receive its own payoff based on the joint



action of all agents involved in the interaction. Formally a  $n$ -player normal-form game  $G$  is a tuple  $\langle N, (A_i), (u_i) \rangle$  where

- $N = \{1, 2, \dots, n\}$  is the set of players.
- $A_i$  is the set of actions available to player  $i \in N$ .
- $u_i$  is the utility function of each player  $i \in N$ , where  $u_i(a_i, a_j)$  corresponds to the payoff player  $i$  receives when the outcome  $(a_i, a_j)$  is achieved.

We call each possible combination of all players' action (joint action) an outcome. There are a number of important solution concepts defined in normal-form games including Nash equilibrium, Pareto-optimality, and social optimality, which can be understood as a subset of outcomes satisfying special properties. These solution concepts also have been adopted as the learning goals when designing different learning strategies in multiagent learning area. In the following, we will introduce each of them in the context of two-player normal-form games which is the most commonly investigated case in the multiagent learning literature. The most important solution concept in game theory is Nash equilibrium. Under a Nash equilibrium, each agent is making its best response to the strategy of the other agent, and thus no agent has the incentive to unilaterally deviate from its current strategy.

**Definition 2.1** A *pure strategy Nash equilibrium* for a two-player normal-form game is a pair of strategies  $(a_1^*, a_2^*)$  such that

1.  $u_1(a_1^*, a_2^*) \geq u_1(a_1, a_2^*), \forall a_1 \in A_1$
2.  $u_2(a_1^*, a_2^*) \geq u_2(a_1^*, a_2), \forall a_2 \in A_2$

For example, considering the two-player normal-form game in Fig. 2.1, we can easily check that there are two pure strategy Nash equilibria in this game:  $(C, D)$  and  $(D, C)$ . Under each of these two outcomes, no agent has the incentive to unilaterally switch its current action to another one.

If the agents are allowed to use mixed strategy, then we can naturally define the concept of *mixed strategy Nash equilibrium* similarly.

**Fig. 2.1** An example of two-player conflicting-interest game

1's payoff, 2's payoff		Player 2's action	
		C	D
Player 1's action	C	0, 0	1, 2
	D	2, 1	0, 0

**Definition 2.2** A *mixed strategy Nash equilibrium* for a two-player normal-form game is a pair of strategies  $(\pi_1^*, \pi_2^*)$  such that

1.  $\bar{U}_1(\pi_1^*, \pi_2^*) \geq \bar{U}_1(\pi_1, \pi_2^*), \forall \pi_1 \in \Pi(A_1)$
2.  $\bar{U}_2(\pi_1^*, \pi_2^*) \geq \bar{U}_2(\pi_1^*, \pi_2), \forall \pi_2 \in \Pi(A_2)$

where  $\bar{U}_i(\pi_1^*, \pi_2^*)$  is player  $i$ 's expected payoff under the strategy profile  $(\pi_1^*, \pi_2^*)$ , and  $\Pi(A_i)$  is the set of probability distributions over player  $i$ 's action space  $A_i$ . A mixed strategy Nash equilibrium  $(\pi_1^*, \pi_2^*)$  is degenerated to a *pure strategy Nash equilibrium* if both  $\pi_1^*$  and  $\pi_2^*$  are pure strategies.

If we again use the same game in Fig. 2.1 as an example, we can verify that there also exists one mixed strategy Nash equilibrium:  $\pi_i^*(C) = \frac{1}{3}, \pi_i^*(D) = \frac{2}{3}$ , for  $i = 1, 2$ .

Another important solution concept in game theory is Pareto-optimality. An outcome is *Pareto-optimal* if and only if there does not exist another outcome under which no player's payoff is decreased and also at least one player's payoff is strictly increased. We can formalize it in the following definition:

**Definition 2.3** An outcome  $s$  is *Pareto-optimal* if and only if there does not exist another outcome  $s'$  such that  $\forall i \in N, u_i(s') \geq u_i(s)$ , and there exists some  $j \in N$ , for which  $u_j(s') > u_j(s)$ .

For example, considering the two-player normal-form game in Fig. 2.1, both the outcomes  $(C, D)$  and  $(D, C)$  are Pareto-optimal.

The last solution concept is social optimality, which refers to those outcomes under which the sum of all agents' payoffs is maximized. For example, considering the two-player normal-form game in Fig. 2.1, both the outcomes  $(C, D)$  and  $(D, C)$  are socially optimal, which coincide with the set of Pareto-optimal outcomes.

## 2.1.2 Repeated Games

Repeated games can be used to model the repeated interactions among the same set of agents for finite or infinite number of times. In repeated games, usually a given normal-form game is played among the same set of players. If a strategic game is infinitely repeated, it is called an infinitely repeated game.

**Definition 2.4** An infinitely repeated game of  $G$  is  $\langle N, (A_i), (u_i), (\succsim_i) \rangle$  where

- $N$  is the set of  $n$  players.
- $A_i$  is the set of actions available to player  $i$ .
- $u_i$  is the payoff function of each player  $i$ , where  $u_i(a_i, \dots, a_n) = p_i$ , the payoff player  $i$  receives when the outcome  $(a_i, \dots, a_n)$  is achieved. Here  $a_i$  is the actions player  $i$  chooses, and  $(a_i, \dots, a_n)$  is called an action profile.

- $\succsim_i$  is the preference relation for player  $i$  which satisfies the following property:  $O_1 \succsim_i O_2$  if and only if  $\lim_{t \rightarrow \infty} \sum_{k=1}^t (p_1^k - p_2^k) / t \geq 0$ , where  $O_1 = (a_{i,t}^1, \dots, a_{n,t}^1)_{t=1}^{\infty}$  and  $O_2 = (a_{i,t}^2, \dots, a_{n,t}^2)_{t=1}^{\infty}$  are the outcomes of the infinitely repeated game, and  $p_1^k$  and  $p_2^k$  are the corresponding payoffs player  $i$  receives in round  $k$  of outcomes  $O_1$  and  $O_2$ , respectively.

The preference relation we adopt here is the limit of means preference relation which is the one we adopt in the book. Notice that there are also other different ways to define preference relation such as discounting and overtaking, and interested reader may refer to the book [1] for details. Also we usually assume that the action set of all agents are the same, i.e.,  $A_1 = \dots = A_n = A$ .

In infinitely repeated games, the strategy of an agent specifies the action choice of the agent for each round. Considering the large space of the agents' possible strategies, it is very difficult for us to identify all the Nash equilibria of a given infinitely repeated game. Thus, usually we turn to characterize the set of all possible payoff profiles that correspond to Nash equilibria of the infinitely repeated game. First, we need to introduce the concepts of *enforceable* and *feasible* payoff profiles which would be useful in the characterization of all possible Nash equilibrium payoff profiles of the infinitely repeated games.

**Definition 2.5** A payoff profile  $(r_1, r_2)$  is enforceable if and only if  $r_i \geq v_i, \forall i \in N$ , where  $v_i$  is agent  $i$ 's minimax payoff.

**Definition 2.6** A payoff profile  $(r_1, r_2)$  is enforceable if and only if there exist nonnegative rational values  $\alpha_a$  such that we have  $\sum_{a \in A} \alpha_a u_i(a) = r_i, \forall i \in N$ , and also  $\sum_{a \in A} \alpha_a = 1$ .

Based on the previous concepts, we can have the following theorem—folk theorem, which characterizes the set of all possible Nash equilibrium payoff profiles of the infinitely repeated games with limit of means criterion.

**Theorem 2.1** *If a payoff profile  $r$  of game  $G$  is both feasible and enforceable, then it is the payoff profile for some Nash equilibrium of the limit-of-means infinitely repeated game of  $G$ .*

## 2.2 Cooperative Multiagent Systems

### 2.2.1 Achieving Nash Equilibrium

Convergence to Nash equilibrium has been the most commonly adopted goal to pursue within different multiagent environments in the multiagent learning literature. We review the representative work within this direction and also point out the limitations.

The interactions among agents are usually modeled as two-player repeated (or stochastic) games. In the work of Claus and Boutilier [2], two different types of