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Lecture Notes in Economics and Mathematical Systems 552

Christopher Suerie

Time Continuity in Discrete Time Models

New Approaches for Production Planning in Process Industries

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Foreword

In recent years great efforts have been made in industry to reduce complexity of production processes and to lower setup times and setup cost. Still, we have observed numerous production facilities where lot-sizing continues to play a major role. Also, the issue of lot-sizing spans a much larger area than merely minimizing the sum of setup and holding costs as it also provides the clue for a better utilization of resources. For example, the author is aware of a case where improved lot-sizing and scheduling increased output by more than 20%!

Still the question remains which lot-sizing model to choose. There is a vast number of lot-sizing models in the literature either based on a discrete time axis or on a continuous times axis. While the former is easier to solve in general aggregation of time often results in missing "optimal" solutions or even feasible solutions (although these might exist). Continuous time models, despite being able to capture more details, often are complex non-linear models resulting in prohibitive computational efforts for its solution.

This was the situation when Christopher Suerie started his PhD project. In the course of the project he came up with a number of excellent ideas to improve modeling capabilities of discrete time model formulations. In the end he has been able to claim that now mixed integer linear model formulations for the *capacitated lot-sizing problem with linked lot sizes* (CLSPL) as well as the *proportional lot-sizing and scheduling problem* (PLSP) exist capturing details that make continuous time model formulations unnecessary. To be more precise, Christopher Suerie has shown how to effectively model restrictions on period overlapping lot sizes (campaigns), namely

- minimal and maximal production amounts,
- minimal resource utilizations throughout campaign production and
- production amounts that are integer multiples of a given batch size.

Furthermore, he has developed a model formulation that mimics period overlapping setup times. He also demonstrates that all his proposals are solvable by state-of-the-art Mixed Integer Programming solvers with rather modest computational efforts – thus making it most appealing for applications in industrial practice.

In the end this PhD thesis not only contributes to a number of single issues that have been treated incorrectly or ineffectively in the literature but provides a comprehensive, unifying modeling framework for single stage lot-sizing and scheduling problems directly applicable in the process industries. It is an excellent piece of research with great potentials for successful applications and worth reading from the first line until the very end.

Hamburg, January 2005 Hartmut Stadtler

Preface

This dissertation is the result of a four-year research effort conducted at the department of Production and Supply Chain Management at Darmstadt University of Technology.

In the beginning of this research we set out to include special characteristics observed in the process industries into mathematical models and algorithms for mid-term production planning. However, after some time I came up looking at a bigger picture. Having analyzed the representation defects of lot-sizing models based on a discrete time scale, I was wondering if it was possible to overcome these defects within this handy time structure. The outcome are mathematical programming model formulations and a temporal decomposition heuristic to model and solve production planning problems of the process industries suffering from the representation defect imposed by time discretization.

This work would not have been made possible without the backing of numerous supporters. First of all, I am deeply indebted to my advisor Professor Dr. Hartmut Stadtler. He not only challenged my efforts by consistently raising new questions, but also encouraged and promoted me as best as possible. As an expert in lot-sizing he was able to provide lots of valuable input in all stages of this dissertation project. Furthermore, I would like to thank Professor Dr. Wolfgang Domschke for his willingness to serve as co-adviser and second referee of this dissertation. His broad expertise and interest in optimization and operations research proved to be a priceless source of information.

Next to my academic advisors, I would like to acknowledge the contribution of my colleagues, who always provided a fruitful working environment and served as interested discussion partners at numerous occasions. Especially, I would like to thank Dr. Jens Rohde for shouldering much of the day-to-day work when I was busy doing research. Moreover, Dr. Gregor Dudek and Martin Albrecht eagerly listened to my new ideas and proofread parts or all of the manuscript. Last but not least I would like to thank Bernd Wagner for providing hardware and software as well as his expertise to conduct some of the computational tests.

However, there is a life besides academia. Nothing of the above would have been materialized without the support of my family. I would like to thank my parents for providing me with the education that enabled me to write this dissertation. Furthermore, I am happy that there is my wife Martina. Although she had to miss myself too often, she backed my efforts during highs and lows. On top of that, she proofread the manuscript several times.

Thanks to all of you.

Darmstadt, January 2005 Christopher Suerie

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List of Abbreviations

APS	Advanced Planning System
BOM	Bill of materials
B&C	Branch and Cut
Cf.	Confer
CLSD	Capacitated Lot-Sizing Problem with Sequence Dependent Setup Costs
CLSP	Capacitated Lot-Sizing Problem
CLSPL	Capacitated Lot-Sizing Problem with Linked Lot Sizes
CP	Constraint Programming
CSLP	Continuous Setup Lot-Sizing Problem
C&B	Cut and Branch
DLSP	Discrete Lot-Sizing and Scheduling Problem
e.g.	exempli gratia (for example)
ELSP	Economic Lot Scheduling Problem
EOQ	Economic Order Quantity
GLSP	General Lot-Sizing and Scheduling Problem
i.e.	id est (that is)
LB	Lower Bound
LB^3	Lower bound after 3 minutes of computational time
LB^5	Lower bound after 5 minutes of computational time
LB^{BEST5}	Lower bound of the best model formulation after 5 minutes of computational time
LB^{LP}	Linear relaxation (after presolve)
LB^{XLP}	Linear relaxation (after presolve and automatic cut generation)
LHS	left-hand side
LP	Linear Programming
med.	medium
MINLP	Mixed Integer Non-Linear Programming
MIP	Mixed Integer Programming
MLCLSP	Multi-Level Capacitated Lot-Sizing Problem
MU	Monetary Units
PLSP	Proportional Lot-Sizing and Scheduling Problem
POST	Period Overlapping Setup Times
RHS	right-hand side
RTN	Resource-Task Network
SPL	Simple Plant Location
SSN	State-Sequence Network

std. dev.	standard deviation
STN	State-Task Network
TBO	Time Between Orders
vi	valid inequalities
w.l.o.g.	without loss of generality

List of Symbols

Indices and index sets:

i	Products or items, $i \in \mathcal{J}$
j	Products or items, $j \in \mathcal{J}$
J	Number of products or items
k	Products or items, $k \in \mathcal{J}$
m	Resources (e.g., personnel, machines, production lines), $m \in \mathcal{M}$
M	Number of resources
s	(Small-bucket) periods, $s \in \mathcal{S}_t$ (except for section 7.5)
s	Periods, $s \in \mathcal{T}^M$ (in section 7.5)
t	Periods, $t \in \mathcal{T}$
T	Number of periods
$\mathcal{J}$	Set of products (1..J)
$\mathcal{J}_m$	Set of products j producible on resource m
$\mathcal{JS}$	Subset of products j, $\mathcal{JS} \subset \mathcal{J}$
$\mathcal{M}$	Set of resources (1..M)
$\mathcal{S}_t$	Set of (small-bucket) periods that form (big-bucket) period t
$\mathcal{T}$	Set of periods (1..T)
$\mathcal{T}^M$	Set of (macro-)periods defined by due dates
$\mathcal{T}_s$	Set of periods which belong to (macro-)period s

Data:

a_j	Capacity consumption to produce one unit of item j (=production coefficient)
a_{mj}	Capacity consumption on resource m to produce one unit of item j (=production coefficient)
$addbat_j$	Maximal number of batches of product j allowed to be bought in T^{fix}
$addcost_j$	Additional cost incurred if one batch of product j is bought in T^{fix}
b_{jt}	Large number, not limiting feasible production quantities of product j in period t
blp_{jt}	Backlog penalty cost associated with one unit of backlog of product j at the end of period t
bs_j	Batch size for lots (campaigns) of product j
c_t	Available capacity in period t
c_{mt}	Available capacity of resource m in period t

d_{jt}	Primary, gross demand for item j in period t (with d_{jT} including final inventory, if given for the planning horizon T)
fc_t	Free capacity in period t based on fixed setup and production decisions
fpc_{jt}	Free capacity in period t to produce product j based on fixed setup and production decisions
h_{jt}	Holding cost for one unit of product j in period t
$maxlot_j$	Maximal lot size (campaign quantity) for product j
$maxlot_{mj}$	Maximal lot size (campaign quantity) for product j on resource m
$maxrate_j$	Maximal production rate of product j
$maxrate_{mj}$	Maximal production rate of product j on resource m
$minlot_j$	Minimal lot size (campaign quantity) for product j
$minlot_{mj}$	Minimal lot size (campaign quantity) for product j on resource m
$minrate_j$	Minimal production rate of product j
$minrate_{mj}$	Minimal production rate of product j on resource m
sc_j	Setup cost for product j
sc_{ij}^{sd}	Sequence dependent setup cost, if a setup operation from product i to product j is performed
sc_{mij}^{sd}	Sequence dependent setup cost, if a setup operation from product i to product j on resource m is performed
sp_j	Maximum number of periods necessary for a setup operation for product j ($sp_j = \lceil st_j / c \rceil + 1$)
sp_{jt}	Maximum number of periods necessary for a setup operation for product j if the setup operation is finished in period t
ssp_{jt}	Safety stock penalty cost associated with the violation of the safety stock target for one unit of product j at the end of period t
sst_j	Safety stock target for product j
st_j	Setup time for product j
st_{ij}^{sd}	Sequence dependent setup time, if a setup operation from product i to product j is performed
tbo_j	Expected time between orders for product j
$util_j$	Minimum utilization rate for product j
x_{jt}^{fix}	Fixed production quantity of product j in period t

Variables:

AB_j	Integer number of additional batches of product j bought in T^{fix}
F_{jt}	Position variable (takes only integer values), the larger F_{jt} the later product j is scheduled in period t
I_{jt}	Inventory of item j at the end of period t
IB_{jt}	Backlog of item j at the end of period t
$ISSV_{jt}$	Safety stock violation for item j at the end of period t
K_{jt}	Campaign variable for product j in period t (current campaign quantity up to period t)
K_{mjt}	Campaign variable for product j on resource m in period t (current campaign quantity up to period t)

KS_{jt}	Cumulated setup time of the current setup operation for product j up to period t [in %]
R_{jt}	Integer number of full batches produced in the current campaign of product j up to period t
S_{jt}	Slack variable, residual quantity of the last batch of product j in period t which is not finished in t
ST_{jt}	Setup time attributed to a setup of product j in period t
ST_{jt}^{b}	Setup time attributed to a setup operation of product j at the beginning or somewhere in period t
ST_{jt}^{e}	Setup time attributed to a setup operation of product j at the end of period t
V_t	Single-product indicator, which indicates that (a) only one product is produced in period t, and (b) the setup state is carried over from period t–1 and carried into the next period t+1 (V_t=1); otherwise (V_t=0)
VV_{jt}	Single-product indicator, which indicates that (a) only product j is produced in period t, and (b) its setup state is brought from period t–1 and carried into period t+1 (VV_{jt}=1); otherwise (VV_{jt}=0)
W_{jt}	Link variables, which indicate that product j was scheduled last in period t–1 and therefore production can continue in period t without performing a new setup operation in period t (W_{jt}=1); otherwise (W_{jt}=0)
X_{jt}	Production quantity of item j in period t (lot size)
X_{jts}	Fraction of demand of item j in period s which is produced in period t
X_{mjt}	Production quantity of product j on resource m in period t (in section 7.5)
X_{jt}^{b}	Production quantity of item j at the beginning of period t (first campaign in t)
X_{jt}^{e}	Production quantity of item j at the end of period t (second campaign in t)
XT_{jt}	Production time of item j in period t
XT_{mjt}	Production time of product j on resource m in period t
XV_{jt}	Production amount of product j in period t, if period t is a period with single-product production
Y_{jt}	Setup variable (=1, if a setup operation for item j is performed (completed) in period t, =0 otherwise)
Y_{mijt}	Setup variable (=1, if a setup operation from item i to item j is performed on resource m in period t, =0 otherwise)
Y_{jt}^{1}	Relative share of setup time for product j in period t (start or within)
Y_{jt}^{2}	Relative share of setup time for product j in period t (end)
Y_{ijt}^{sd}	Sequence dependent setup variable (=1, if a setup operation from item i to item j is performed in period t, =0 otherwise)
YI_t	Setup operation indicator for period t (=1, if a setup operation occurs in period t, =0 otherwise)
Z_{jt}	Setup state variable (=1, if item j is setup at the end of period t, =0 otherwise)

Z_{0t}	Setup state variable, indicates whether any setup state persists at the end of period t (=0) or none (=1). The latter case indicates that a setup operation for product is going on at the end of period t
Z_{mjt}	Setup state variable (=1, if item j is set up on resource m at the end of period t, =0 otherwise)
ZS_{jt}	Binary setup operation state variable (=1, if a setup operation for product j is going on at the end of period t (and can continue in period t+1), =0 otherwise)

Parameters of the temporal decomposition heuristic:

Δ	Length of the rolling window
Φ	Overlap of rolling windows of two consecutive planning steps
Ψ	Number of periods with relaxed integrality constraints at the end of each (except for the last) rolling window
fix	Production quantities are fixed in the time interval preceding the rolling window
max	Setup operations in the time interval following the rolling window are anticipated by their maximum
mean	Setup operations in the time interval following the rolling window are anticipated by their mean
min	Setup operations in the time interval following the rolling window are not anticipated
T^{fix}	Number of periods preceding the rolling window (= number of periods with fixed setup decision)
var	Production quantities are allowed to vary in the time interval preceding the rolling window

1 Introduction

1.1 Motivation

Different modeling paradigms often collide at the interface of short-term, operational production planning and mid-term production planning. Mid-term plans are most often based on a discrete time scale made of weekly or monthly buckets without too much detail. On the other hand, short-term operational planning needs a lot more detail and therefore comprises time buckets with the size of days or shifts – or even better – is not attached to a fixed grid of time buckets, that is a continuous time scale.

Both paradigms have their legitimacy in their respective settings. For mid-term planning it is sufficient to know, that e.g. products A, B and C will be produced in the quantities 50, 80 and 30 units in week 17. On the other hand, it is important to know that the setup change from product A to B for the stamping machine needs to take place on e.g. Tuesday between 2.30 p.m. and 5 p.m., because setup personnel has to be scheduled for this event.

Models and algorithms for both, production planning on a discrete time scale and for production planning on a continuous time scale, are known in large numbers. A missing link and the focus of this thesis will be the representation of arbitrary (continuous) plans on a discrete time scale.

From a theoretical point of view this idea is very appealing, as it would allow to combine short-term and mid-term planning into one modeling approach. If a telescopic time scale with shorter time buckets at the beginning, to capture the detail necessary for short-term production planning, and bigger time buckets towards the end of the planning horizon is used, both planning steps can be accomplished with only one model. As a consequence, the structural differences which often complicate communication at the interface of short-term and mid-term production planning are reduced.

Anyhow, not a global model that solves all kinds of production planning problems will be presented here, but rather several important building blocks, primarily intended for mid-term production planning and thus bucket-oriented will be introduced. These building blocks may be used as different extensions to standard lot-sizing models. They are motivated by practical production planning problems. Moreover, built together into one model, it will be possible to represent arbitrary continuous production plans in a bucket-oriented setting.

The application of these planning models, which first comes into mind, is process industries. Furthermore, also discrete production environments might be eligible for use of at least some of the building blocks that will be presented. This stems not only from the analogy between discrete production and process industries,[1] but can also be seen from the case descriptions which follow in section 1.3.

1.2 Some Definitions

In different industries different terms are used – from a planning point of view – in the same or similar context. Here the focus will be on the terms "lot-sizing" and "campaign planning" first, which are in fact terms originating from totally different sources.

The term "lot-sizing" has its roots in a discrete production environment. Lot-sizing is the arrangement of demands for the same product in different periods to a single production order ("lot").[2] This means, that customer orders (or anonymous demand) with different due dates for a certain product are combined to form a production order. The reasoning is, that each production order is usually associated with a certain fixed cost (setup cost). If customer orders were produced as demanded ("lot-for-lot"-production), this would strongly affect costs. Furthermore, it would affect capacity, because setups will generally consume also a fixed amount of capacity. To avoid that, customer orders are combined. The result of lot-sizing is a production plan, which shows when to produce (e.g., in week 13) and how much (e.g., 100 units).

The term "campaign planning" on the other hand is typically used in the process industries. There, two variants of campaigns have to be distinguished: single-product campaigns and multi-product campaigns.[3] In analogy to lot-sizing a single-product-campaign can be defined as a production order, which comprises several customer orders (or anonymous demand) that share an unique setup state. With respect to the production environment there may be several specialties or additional constraints. The most important one is, that the production order may be made of several batches, with a batch defined as a combination of a production amount and a certain task.[4] The batch size is often fixed and determined by the size of the production resource (e.g., a tank). Anyhow, in general – at least at this level of abstraction – there is not a big difference between a lot in lot-sizing and a single-product campaign in campaign planning.

Multi-product campaigns do not fit into the lot-sizing scheme as easily. Here, a campaign consists of several products requiring different setup states, but campaigns are built such, that setup operations within a campaign require much less effort (cost and/or time) than setup operations between campaigns.[5] An analogy in lot-sizing is the grouping of products into families such that only minor setups are

1 Cf. Voß and Witt (2003) pp. 75-81.

2 E.g., Chase et al. (1998) pp. 648-649, Günther and Tempelmeier (1997) p. 182, Gutenberg (1983) p. 201 and Silver et al. (1998) p. 198.

3 Cf. Blömer (1999) p. 16 and Overfeld (1990) pp. 87-88.

4 Cf. Schwindt and Trautmann (2000) p. 502.

5 Cf. Overfeld (1990) pp. 87-88.

necessary between members of a product family and major setups are necessary between families.[6]

Moreover, a term used in discrete production as well as in the process industries is "batching". Unfortunately the meaning is different in both contexts. In the process industries a batch is defined as a combination of a production amount and a certain task. If the batch size is not determined by the production resources, the decision on batch sizes is called batching here (first step).[7] In a second step, batches requiring the same resource configuration (setup) are combined to form a campaign. The reason why production planning in the process industries is often in batches and the batches are not put together to form a bigger batch for planning purposes is, that resources or tanks often limit certain production tasks.[8] In discrete production however, the second step is referred to as batching. Here, the combination of production orders belonging to the same order family is called batching.[9] For a more extensive discussion we refer to Voß and Witt (2003), who discuss the different meanings of batching in discrete production and process industries as well as find and define analogous terms in these two fields.[10]

In the remainder of this thesis we will stick to the following nomenclature:

- A lot (or lot size) is the production amount of a production order which is produced in one production run without changing the setup state.
- As there are only settings in the scope of this thesis which require the planning of single-product campaigns, the term "campaign" can be used as a synonym to the term "lot".

Anyhow, when literature from the different fields is discussed in chapters 2 and 4, the term "lot" will be used, if the lot size is produced within a period (time bucket), whereas the term "campaign" will be used, if the lot extends over several periods. But this is only not to confuse readers familiar with only one field and one should keep in mind, that generally – at this level of abstraction – there is no big difference between lot-sizing and campaign planning, apart from the latter requiring some side constraints.

Furthermore, we will refrain from using the term "batching" to avoid confusion of the reader, because this term – as mentioned above – has very different meanings in discrete production and in the process industries.

1.3 Case Studies

The production planning models studied in this thesis are not only interesting from a theoretical point of view, but also relevant from a practitioner's perspective, as the following case descriptions illustrate.

6 Cf. Potts and van Wassenhove (1992) p. 397.
7 Cf. Trautmann (2001) p. 5.
8 Cf. Voß and Witt (2003) pp. 76-77, 81.
9 Cf. Potts and Kovalyov (2000) pp. 228, 231 and Voß and Witt (2003) pp. 78, 81.
10 Cf. Voß and Witt (2003) pp. 75-81.

- *Napkin production*
  The production process consists of three stages. At the first stage paper is produced in a continuous process. The second processing step – and in this case the bottleneck – is the conversion of paper into napkins. Here, an emblem or design is printed on the paper, which is then folded into shape. The folding operation involves a difficult setup step, which takes up to 36 hours. At the third stage the folded napkins are wrapped and packaged.[11]

  Although production plans in this case assume a period length of approximately one month, setup operations consume a substantial portion of time (5 % of capacity) and therefore need to be accounted for as accurately as possible. Otherwise production capacity may be lost or orders that should have been taken are declined.
- *Self-adhesive laminate*
  Self-adhesive laminates comprise of two layers. The top layer is formed by the face material made of paper or a synthetic, which is usually coated. On the back-side of the top layer an adhesive is applied. The bottom layer is mostly made of paper, which is silicon coated for easier release of the top layer.[12]

  In this case the bottleneck to be planned for is the coating of the face material. The planning horizon is three weeks and the varnish/paste coater is utilized five days per week and 24 hours per day. The period length is one day and the setup time is about 5 % of capacity. It is not only important to account for setup times correctly because of the tight capacity situation, but also because setups waste energy and raw materials, which go into scrap.[13]
- *Production in a chemical plant I*
  A chemical plant is considered in this case. Here, a reactor has to be planned for. This reactor can operate in different modes, producing exactly one distinct product in each mode. Changeovers are not only very costly, but also consume a considerable portion of available capacity. The planning horizon comprises one to three years with monthly buckets. Production plans are only accepted by the planners, if they meet certain criteria. These are e.g. that campaigns have to obey a minimal size of 300 tons or that they have to be built of batches with a size of 100 tons each.[14]
- *Production in a chemical plant II*
  Here, a continuous process in a chemical plant that operates 365 days per year and 24 hours per day is to be planned for. The process is interrupted only for maintenance purposes a few days each year. In this process the minimization of changeovers is of paramount importance, because the plant produces off-grade products for a few days after each changeover between two products. Therefore, a minimum length is imposed on each production run. On the other hand,

[11] Cf. Gopalakrishnan et al. (1995) p. 1974.
[12] Cf. Raflatac (2003).
[13] Cf. Porkka (2000) pp. 7, 51-57.
[14] Cf. Kallrath and Wilson (1997) pp. 303-325 and Kallrath (1999) p. 335.

storage space is limited and costly to increase. Furthermore, the process requires that the plant is always run at a minimum utilization rate.[15]

- *Campaign planning*
  A software company providing supply chain planning software wanted to enhance the modeling and solution capabilities of its mid-term production planning module. In this case it is crucial not only to solve a special case, but to come up with an universal model/algorithm that fits into the architecture of the software system in place. Characteristics within the scope of this project have been an exact modeling of setup operations within a bucket-oriented time structure, specification of minimal campaign lengths and planning of campaigns consisting of batches with fixed size.

1.4 Outline of Thesis

This thesis is organized as follows. In the second chapter basic models in lot-sizing are introduced. According to their underlying time structure they are classified into big-bucket, small-bucket and hybrid models. After having studied the differences of these models in detail, the third chapter analyzes their representation defects with respect to a continuous time scale. Thus, the effect discretization of time imposes on plans, that can be generated by those basic lot-sizing models, is evaluated. This analysis is based on four cornerstones, which are the representation of setup states, the representation of lot sizes, the representation of setup operations and different assumptions on resource utilization.

The fourth chapter provides a thorough literature review which is divided into two parts. The first part reviews basic models in lot-sizing introduced in the second chapter with special emphasis on the extensions to model time continuity as defined in chapter three. The second part contains a review of model formulations originating from applications in the process industries. These often incorporate some aspects of time continuity. As some of these model formulations are based on a continuous time scale, this second part of the literature review is further divided into model formulations based on a discrete representation of time and those based on a continuous representation of time.

The planning framework and techniques considered capable of solving the later proposed model formulations are presented in chapter five. As solution techniques mathematical programming and decomposition will be introduced.

The sixth chapter contains the heart of this thesis. Here, the modeling and solution approach is presented. Mathematical programming model formulations are provided for the four aforementioned aspects to model time continuity in a time-indexed setting (setup states, lot sizes, setup operations and resource utilization). These extensions are given for two different basic models. Furthermore, they are provided as building blocks and may be freely combined dependent on the actual

[15] Cf. Lee and Chen (2002) pp. 16-17.

decision situation. Finally, a decomposition heuristic is proposed to allow also for the solution of problems of bigger size.

Computational results are provided in the seventh chapter. These are again organized based on the four aspects to model time continuity in a time-indexed setting (setup states, lot sizes, setup operations and resource utilization). Solutions are analyzed to give insights into what makes certain decision situations difficult. Moreover, computational performance of the proposed model formulations is assessed by comparing them to other model formulations from literature. The extensibility of the proposed model formulations is shown as well as their independence from solver technology.

Finally, chapter eight summarizes the achievements of this thesis and gives a brief outlook on further research opportunities.

2 Basic Models in Lot-Sizing

2.1 Classification of Lot-Sizing Models

Obtaining cost-efficient production plans balancing the trade-off between setup and inventory holding costs – lot-sizing – has been a fundamental goal of practitioners since the beginning of industrialization. The first published work in this area by Harris titled "How many parts to make at once?" dates back as far as to 1913.[16] Since then, a broad stream of research has been developed, dealing with various types of lot-sizing problems for many different applications.

These can be classified according to different attributes.[17] For ease of presentation these attributes are clustered into three sets according to their main relationship: time, resource and product.

The first set "time" contains all attributes with relations to the time structure of the model and the data used:

- *Planning horizon*: The planning horizon may be finite or infinite. Models with an infinite planning horizon usually assume a constant demand rate like the Harris' economic order quantity (EOQ)-model[18] and will not be considered in the remainder.
- *Time scale*: The time scale may either be continuous or discrete. If a discrete time scale is chosen, time buckets may be either big or small and either uniform or non-uniform. Most standard lot-sizing models assume a uniform time discretization, which means that all time buckets have the same size (see sections 2.2 and 2.3). Nevertheless, sometimes a telescopic time scale is chosen with larger time buckets towards the end of the planning horizon[19] or time buckets may be non-uniform for other reasons[20]. The distinction into small- and big-bucket models concerns the relative length of the time periods with respect to the ex-

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16 Harris (1913).

17 Other compilations of attributes and classifications of lot-sizing models and literature can be found in e.g., Derstroff (1995) pp. 20-24, Domschke et al. (1997) pp. 69-75, Haase (1994) pp. 3-7, Karimi et al. (2003) pp. 366-367, Kuik et al. (1994) pp. 247-249, Meyr (1999) pp. 46-55, Salomon (1991) pp. 21-22, Stadtler (2001) pp. 39-40 and Wolsey (2002) pp. 1589, 1591, 1595.

18 Cf. Harris (1913).

19 E.g., Timpe and Kallrath (2000) pp. 424-425.

20 E.g., Karimi and McDonald (1997) p. 2702.

pected length of any individual production lot.[21] In models with small time buckets it is usually assumed that in each period only one or at most two products may be produced. Therefore small-bucket models integrate lot-sizing and scheduling by not only determining the lot sizes, but also the sequence of production orders. On the other hand, big-bucket models permit multiple products to be produced each period without making any assertion about the sequence of orders.

- *Temporal development of parameters/data*: Parameters can either vary over time (dynamic) or not (static). Often models are distinguished into dynamic or static lot-sizing models according to the temporal development of demands,[22] but generally all parameters (e.g., production capacity, cost parameters, production coefficients) may vary over time.
- *Availability and knowledge of parameters/data*: With respect to the availability and knowledge of the problem data deterministic and stochastic models have to be distinguished. In deterministic models all parameters and data are assumed to be known prior to planning. Stochastic models on the other side try to incorporate the uncertainty of the future into the planning model. This is usually done by assuming a certain distribution or range of values instead of a distinct value for a certain parameter. Typical parameters which are modeled stochastically are e.g. external demands or quantities of defective items.[23] Only deterministic problems will be discussed in the remainder.
- *Objective function*: Most commonly the objective of a lot-sizing problem is to minimize the sum of several cost components. Nevertheless, sometimes other objective functions are defined.[24] These can be either monetary like the maximization of profits or sales, or non-monetary. Then, the goal is not transformed into monetary units, because it is a rather physical accomplishment (e.g., resource leveling) or a temporal objective (e.g., minimization of maximum lateness or total completion time)[25].
- *Cost components*: As stated above the standard objective function is the minimization of several cost components. These comprise classically inventory holding costs and setup costs.

  Inventory holding costs are typically taken into consideration as a linear cost function of the quantity of products in stock at certain points in time. Economically they mainly consist of the costs of capital tied up in inventory. Other parts included in inventory holding costs are costs associated with warehouse operations, taxes, insurance premiums, obsolescence and shrinkage.[26]

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21 Cf. Salomon (1991) p. 21. Buckets in small-bucket models can have a considerable length depending on the industry and level of aggregation. E.g., De Matta and Guignard (1994) discuss an example of a small-bucket model with a bucket length of one week.

22 Cf. Domschke et al. (1997) p. 70, Kuik et al. (1994) p. 247 and Salomon (1991) p. 21.

23 Cf. Haase (1994) p. 3.

24 Cf. Domschke et al. (1997) p. 73 and Kallrath (2002b) p. 224.

25 Cf. Potts and van Wassenhove (1992) p. 398.

26 Cf. Derstroff (1995) p. 23 and Haase (1994) p. 5.

Setup costs are costs incurred by the production process. Whenever a lot of any product is produced, resources involved in the production process have to be set up to cope with that specific product, e.g., re-tooling of a machine. These costs are charged to the objective function as setup costs. Setup costs consist of direct costs (e.g., cleaning materials) and opportunity costs. Opportunity costs are charged, if setup times, that are the times associated with setup operations, are not considered in the model explicitly. Then, one has to figure out how much capacity has been lost due to the setup operation in order to determine the value of this lost capacity (e.g., contribution margin of those products that could have been produced during the setup operation). Of course, these opportunity costs are hard to estimate as they depend strongly on the scarcity of available capacity which may vary over time and which often is known only after lot-sizing has been done. Therefore, many authors recommend to include setup times into the model and to charge only direct setup costs in the objective function.[27]

Besides these classical cost components of lot-sizing problems other cost components might be considered in the objective function.

First, there are more types of setup related costs. Reservation costs might be charged, if there is a cost associated with preserving the current setup state, when there is no production. Switch-off costs might replace setup costs, if the costs associated with a specific production lot are related primarily to the end of the production process and not to the start (e.g., the main cost component results from cleaning). Generally, it suffices to include either setup or switch-off costs as long as the costs are assumed constant over time and no net present value calculation is performed in the objective function. Furthermore, Wolsey (2002) distinguishes between start-up costs and setup costs, where start-up costs are the costs associated with the start of a production lot and setup costs are charged in each period of production (including the start-up period).[28]

Second, there are also more inventory related costs. These deal with the case, that there is not enough inventory to meet demand. In this case the demand is either lost (lost sales) or fulfilled in later periods (backlog). Both cases are not desirable and therefore a penalty cost is usually associated with these types of "negative" inventories.

Moreover, production costs are most often assumed constant over time and therefore irrelevant in this decision situation. Nonetheless, it might be economically correct to assume declining production costs.[29]

Finally, overtime costs for using extra capacity might be considered in the objective function.

The second set of attributes concerns mainly the "resources" involved in the production planning problem.

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27 Cf. Kuik et al. (1994) pp. 249-250 for more criticism to this approach.

28 Cf. Wolsey (2002) p. 1597.

29 Cf. Domschke et al. (1997) p. 72.

- *Capacities*: Capacities of resources can be assumed finite (capacitated) or infinite (uncapacitated). If assumed finite, they might be extended by overtime at a certain cost. Again, this extension can be finite or infinite. Usually capacities may be used up to a fixed budget in each period, e.g., according to a working calendar. On the other hand, very rarely, resources are assumed to be not renewable or only partially renewable. This means the resource availability in a certain period depends on the use of that specific resource in former periods.[30]
- *Product-resource-assignments*: Product-resource-assignments[31] are either such, that each product is assigned to only a single resource (unique assignment) or not.
- *Number of resources*: The planning problem may comprise of one or more resources. If a specific operation can be performed on more than one resource, these resources are called parallel resources. They can be either identical (with respect to the production coefficients, capacities and sets of possible product-resource-assignments) or not. On the other hand, if a single operation requires two or more resources in parallel (e.g., a machine and a worker), we will talk of a problem with multiple resources.
- *Product/operation structure*: The product/operation structure[32] which shows the flow of materials through the production system may be either cyclic or non-cyclic. The product/operation structure is deemed cyclic, if at least one end-product requires two operations at the same resource.
- *Minimal utilization rates*: Minimal production / utilization rates are sometimes taken into account.[33] They are necessary to avoid production plans in which resources are utilized only to a negligible extent. In that case it might be more economical to turn this resource off and shift production to another resource or period.
- *Production coefficient*: Production coefficients are usually deemed constant. That means a production function of Leontief type[34] or linear technology is assumed as a basis. Changes in intensity as considered in the Gutenberg production function[35] are regularly not taken into account, but sometimes the assump-

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30 Cf. Kimms (1997) pp. 66-68 for an example with partially renewable resources.

31 In this context of planning it is generally not sufficient to examine products at this level of detail. Instead operations should be focused on here, because an operation uses part of the available resource capacity, while a product is usually treated by several operations on (possibly) different resources (e.g., Tempelmeier (2003) p. 207). Nevertheless, we will keep this distinction in mind, but continue to use the terms "product" and "item" as synonyms as done in most of the lot-sizing literature.

32 Cf. Tempelmeier and Helber (1994) pp. 297-298 and Tempelmeier and Derstroff (1996) p. 739.

33 E.g., Kallrath and Wilson (1997) p. 315, Lee and Chen (2002) pp. 21-22 and Wolsey (2002) p. 1597.

34 Cf. Domschke and Scholl (2000) p. 89.

35 Cf. Domschke and Scholl (2000) pp. 92-95, Thommen (1991) pp. 404-407 and Wöhe (1990) pp. 587-594.